Progressive Education Society's

Modern college of Arts, Science & Commerce, Ganeshkhind, Pune 16 (Autonomous)

End Semester Examination Oct/Nov 2024

Faculty: Science and Technology

Program: BScGen03 **Semester V** **SET A**

Program(Specific): B.Sc **Course Type:Core**

Class: T.Y.B.Sc.(Mathematics) **Max. Marks:35**

Name of the Course: Metric Spaces **Course Code: 24-MT 351**

Paper No: I **Time: 2 Hours**

Instructions To the Candidates:

1. There are 3 sections in the question paper. Write each section on separate page.
2. All Sections are compulsory.
3. Figures to the right indicate full marks.
4. Draw a well labelled diagram wherever necessary.

SECTION: A

Q.1) Attempt any **five** of the following. [10 marks]

a) Does $d(x, y) = |\sin(x - y)|$ define a metric on $\mathbb{R}$? Justify.

b) Let X be the discrete metric space and $x \in X$. Find i) B(x,1/2), ii) B(x,2).

c) In $\mathbb{R}$ with usual metric, write closure of i) (0,1], ii) $\mathbb{Q}$.

d) Show that $(0, 1]$ and $[1, \infty)$ are homeomorphic.

e) Give an example of subset of $\mathbb{R}$ which is connected but not compact.

f) Find interior of the sets i) [1,2), ii) $\mathbb{Z}$ in $\mathbb{R}$ with usual metric.

g) Is arbitrary intersection of open sets open in a metric space? Justify.

SECTION: B

Q.2) Attempt any **three** of the following. [15 marks]

a) Show that the following set is closed in $\mathbb{R}^2$ with usual metric.

$$S = \{(x, y) \in \mathbb{R}^2 \mid xy = 0 \}$$

b) If A and B are any subsets of a metric space X.
Show that $\overline{A \bigcup B} = \overline{A} \bigcup \overline{B}$

c) Prove that limit of a sequence in a metric space (X,d) is unique.

d) Let (X,d) be a metric space. Show that every convergent sequence in X is a Cauchy sequence.

e) Let A and B be two connected subsets of X with the property that $A \bigcap B \neq \phi$ then prove that $A \bigcup B$ is connected

SECTION: C

Q.3) Attempt any **one** of the following. [10 marks]

a) Let (X, d) be a metric space. Define

$$\delta(x, y) = \frac{d(x, y)}{1 + d(x, y)}, \quad \forall x, y \in X$$

Show that δ is a metric on X.

b) Define compact subset in a metric space. Show that if A and B are compact then $A \bigcup B$ and $A \bigcap B$ are compact.

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